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Determination of $\sigma(e^+e^- \to \pi^+\pi^-)$ from radiative processes at DAΦNE

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Abstract. We have measured the cross section $\sigma(e^+e^-\to\pi^+\pi^-\gamma)$ with the KLOE detector at DAΦNE, at an energy $W = M_\phi = 1.02$ GeV. From the dependence of the cross section on $m(\pi^+\pi^-) = \sqrt{W^2 - 2WE_\gamma}$, where E_γ is the energy of the photon radiated from the initial state, we extract $\sigma(e^+e^-\to\pi^+\pi^-)$ for the mass range $0.35 < m^2(\pi^+\pi^-) < 0.95$ GeV^2. From our result we extract the pion form factor and the hadronic contribution to the muon anomaly, a_μ.

1 Hadronic cross section at DA*Φ*NE

1.1 Motivation

The recent precision measurement of the muon anomaly a_μ at the Brookhaven National Laboratory [1] has led to renewed interest in accurate measurements of the cross section for e^+e^- annihilation into hadrons. Contributions to the photon spectral functions due to quark loops, are not calculable for low hadronic mass states because of the failure of perturbative QCD in such conditions. A very clean way out has been known for a long time [2]. The imaginary part of the hadronic piece of the spectral function is connected by unitarity to the cross section for $e^+e^-\to$hadrons. A dispersion relation can thus be derived, giving the contribution to a_μ as an integral over the hadronic cross section multiplied by an appropriate kernel. An example of a first complete estimate of the correction was given in 1985, $\delta a_\mu^{\rm had}$=707(6)(17) $\times 10^{-10}$ [3]. The process $e^+e^-\to\pi^+\pi^-$ contributes ∼500 out of the ∼700 value above. The cross section for $e^+e^-\to\pi^+\pi^-$ becomes negligible above 1 GeV.

The most recent measurements of $\sigma(e^+e^-\to\pi^+\pi^-)$ in the energy interval 610$< m(\pi^+\pi^-) <$961 MeV come from CMD-2 at Novosibirsk. They claim a systematic error of 0.6% with a statistical error of ∼0.7% [4]. Their data have been used most recently together with τ and e^+e^- data up to 3 GeV, in an attempt to produce a firm prediction for comparison with the BNL result [5]. There is unfortunately a rather strong disagreement between the $\delta a_\mu^{\rm had}$ value obtained using $e^+e^-\to\pi^+\pi^-$ data and τ decay data, after isospin corrections. Finally the $e^+e^-\to\pi^+\pi^-$ based result disagrees by ∼3 σ with the BNL [1] measurements. There are thus many reasons for new measurements of the e^+e^- annihilation cross section into two pions.

1.2 Initial state radiation

Initial state radiation, ISR, is a convenient mechanism which allows studying $e^+e^-\to$ hadrons, over the entire energy range from $2m_\pi$ to W, the center of mass energy of the collision. In the case of interest it is potentially vitiated by the possibility of final state radiation. For a photon radiated before annihilation of the e^+e^- pair, the $\pi^+\pi^-$ system energy is $m(\pi^+\pi^-) = \sqrt{W^2 - 2WE_\gamma}$, thus one measures the coupling to pions of an off-mass shell photon of mass m$(\pi^+\pi^-)$. For a photon radiated by the final state pions, the virtual photon coupling to the $\pi^+\pi^-$ pair has a mass W. By just counting powers of α, the

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relative probability of ISR and FSR are equal. This requires very careful estimates of the two processes in order to be able to use the reaction $e^+e^-\to\pi^+\pi^-\gamma$ to extract $\sigma(e^+e^-\to\pi^+\pi^-)$. The Karlsruhe group has given much attention to this problem and has also developed the EVA and PHOKHARA Monte Carlo programs [6] - [10] which are fundamental to our analysis. In the following we will refer to the invariant mass squared of the $\pi^+\pi^-$ system as s_π. In general the $\pi^+\pi^-\gamma$ and $\pi^+\pi^-$ cross section are related through:

$$s_\pi \frac{\mathrm{d}\sigma(\pi^+\pi^-\gamma)}{\mathrm{d}s_\pi\,\mathrm{d}\cos\theta} = \sigma(\pi^+\pi^-,\ s_\pi) \times F(s_\pi,\ \theta_\gamma)$$

Integrating the cross section above, for $0 < \theta_\gamma < \overline{\theta}$ and $180° - \overline{\theta} < \theta_\gamma < 180$ we get

$$s_\pi \frac{\mathrm{d}\sigma(\pi^+\pi^-\gamma)}{\mathrm{d}s_\pi} = \sigma(\pi^+\pi^-,\ s_\pi) \times H(s_\pi,\ \overline{\theta}) \qquad (1)$$

Equation 1 defines the radiator function $H(s_\pi,\ \overline{\theta})$. In the following we will drop the variable $\overline{\theta}$ which is a constant in the present work. The radiator function H used in our analysis is obtained from the PHOKHARA Monte Carlo program. Our present analysis is based on the observation of [6], that for small polar angle of the radiated photon, the ISR process vastly dominates over the FSR process. At lowest order the $\pi^+\pi^-\gamma$ cross section diverges as $1/\sin^2\theta$, just like $\sigma(e^+e^-\to\gamma\gamma)$. We limit ourselves in the following to studying the reaction $e^+e^-\to\pi^+\pi^-\gamma$ with $\theta_\gamma < 15°$ or $\theta_\gamma > 165°$. For small $m(\pi^+\pi^-)$, the di-pion system recoiling against a small angle photon will results in one or both pions being lost also at small angle. We are therefore limited to measuring $\sigma(\pi^+\pi^-)$ for $m(\pi^+\pi^-)$ >550 MeV. We will be able to investigate the cross section near threshold, as soon as next to leading order calculation for FSR become available. This is of great importance, since there are no recent, good measurements of $\sigma(\pi^+\pi^-)$ at low mass, which weigh strongly in the estimate of $\delta a_\mu^{\mathrm{had}}$.

2 Event selection and analysis

Details on event selection and analysis are reported in [11].

2.1 Fiducial volume

The fiducial volume for $\pi^+\pi^-\gamma$ events has been chosen in order to maximize the acceptance for $\pi^+\pi^-\gamma$ events due to ISR. The main background processes are $e^+e^-\to\phi\to\pi^+\pi^-\pi^0$ and radiative Bhabha scattering. At low mass the final state $\mu^+\mu^-\gamma$ becomes important but requiring small θ_γ removes most of this final state. We rely otherwise on kinematics and particle identification, ID, in order to reject the mentioned backgrounds to a negligible level. From studies with EVA, confirmed with PHOKHARA, a good compromise for the fiducial volume is $|\cos\theta_\gamma| > \cos 15°$ and $50° < \theta_{+-} < 130$ for the polar angle of both tracks at the interaction point. These requirements ensure abundant statistics for the measurement and contamination of FSR hard radiation well below the 1% level. The efficiency for the events falling inside the fiducial cuts is contained in the value of the radiator, obtained from PHOKHARA. We note that no detected photon is required, nor are events rejected if photons are present.

Table 1. List of systematic uncertainties from the three sources: experimental, theoretical, and ignoring FSR

Acceptance	0.3%
Trigger	0.6%
Tracking	0.3%
Vertex	1.0 %
Likelihood	0.1 %
Track Mass	0.2 %
Background subtraction	0.5 %
Unfolding	0.6 %
Total exp systematics	1.4 %
luminosity	0.6 %
Vacuum Polarization	0.2 %
Total theor. systematics	0.7 %
FSR resummation	2.0 %

2.2 Efficiency

The contribution of the systematic error for each efficiency to the total error is shown in Table 1.

3 Results

The cross section for $e^+e^- \to \pi^+\pi^-\gamma$, after applying the corrections described above, is shown in Fig. 1. In order to get the $e^+e^- \to \pi^+\pi^-$ cross section, the radiator function $H(s_\pi, \overline{\theta}_\gamma)$ is needed. The radiator is obtained from PHOKHARA, setting $F_\pi(s_\pi) = 1$ and *switching off* the vacuum polarization of the intermediate photon in the generator. The radiator $H(s_\pi, \overline{\theta}_\gamma = 15°)$ is also shown in Fig. 1, left.

3.1 FSR and vacuum polarization corrections

Part of the multiphoton events are removed by the m_{trk} cut. The effect of this cut is evaluated using PHOKHARA 1.0 which, however, does not include soft radiation in the final state accompanying one hard photon from the initial state. An preliminary estimate done with PHOKHARA-II, shows that this effect is smaller than 2%. We *do not correct* here for FSR photons and we include in the theoretical systematic error the full value (2%) due to this effect. In order to obtain the pion form factor, or the *bare*

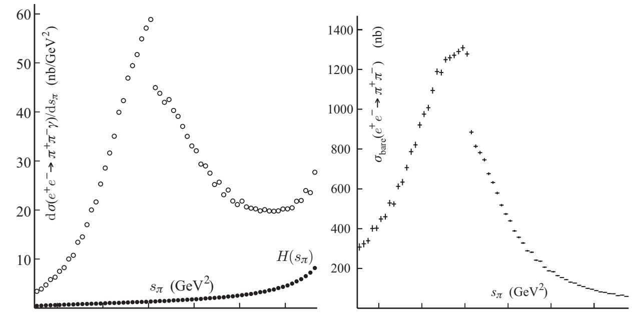

Fig. 1. *Left.* Cross section for $e^+e^- \to \pi^+\pi^-\gamma$. The radiator $H(s_\pi, \bar{\theta}_\gamma)$, is also shown. *Right.* Bare cross section for $e^+e^- \to \pi^+\pi^-$

cross section, vacuum polarization effects must be subtracted. This can be done by correcting the cross section for the running of α, see [12,5], as follows:

$$\sigma_{\text{bare}} = \sigma_{\text{dressed}} \left(\frac{\alpha(0)}{\alpha(s)} \right)^2 \tag{2}$$

We have used an approximate, but quite adequate for the purpose, estimate of $\alpha(s)$ [13]. Fig. 1, right, shows the resulting cross section.

3.2 Results and comparisons with existing data

We have used the *bare* cross section to evaluate of $\delta a_\mu^{\text{had}}$ in the region covered by the CMD-2 experiment ($0.37 < s_\pi < 0.93$). The resulting value (in 10^{-10} units) is :

$$\delta a_\mu^{\text{had}} = 374.1 \pm 1.1_{\text{stat}} \pm 5.2_{\text{syst}} \pm 2.6_{\text{theo}} \ {}^{+7.5}_{-0.0}\big|_{\text{FSR}} \tag{3}$$

to be compared with:

$$\delta a_\mu^{\text{had}}(\text{CMD-2}) = 368.1 \pm 2.6_{\text{stat}} \pm 2.2_{\text{syst+theo}} \tag{4}$$

The statistical error is negligible. The current systematic error amounts to 1.4%, but there is still room to improve it down to $\simeq 1\%$. Clearly the dominating error is the one we have conservatively assigned to the FSR radiation effect. This error will be substantially reduced once the new PHOKHARA version will be inserted into the KLOE MC.

The central value of our result in the region $0.37 < s_\pi < 0.93$ is slightly larger than the one obtained by CMD-2. The discrepancy is, at the moment, not significant (0.5 σ). This difference is, however, not equally distributed in s_π, as summarized below.[1]

s_π, GeV^2	δa_μ, KLOE	δa_μ, CMD-2
$0.37 - 0.60$	$256.2 \pm 4.1(^{+5.1}_{-0}$ FSR)	249.7 ± 2.2
$0.60 - 0.93$	$117.9 \pm 2.1(^{+2.3}_{-0}$ FSR)	119.8 ± 1.1

Our data differ from the CMD-2 results mostly below the ρ-peak. However, for the mass squared range $0.6 < s_\pi < 0.9$, our data confirm the discrepancy between τ data and $e^+e^- \to \pi^+\pi^-$ results, which is $\sim$10-15%.

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[1] This is our evaluation of a_μ^{had}(CMD-2), based on the values tabulated in [4]